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HEAT EXCHANGE ON A THERMAL INITIAL SECTION IN THE EVAPORATION OF TURBULENT FALLING LIQUID FILMS

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An approximate analytical solution is obtained for a problem of convective heat transfer on a thermal initial section in the evaporation of turbulent falling liquid films.

The evaporation of falling liquid films finds application in evaporators, refrigeration systems, and various types of water distillation units [1]. The high rate of the process at low temperature heads makes it possible to process solutions of thermolabile substances, and in many cases it is the only method of doing so. It is also best to minimize the time in the heating zone for such substances, which makes it necessary to use relatively short heat-exchange surfaces. The effect of the thermal initial section becomes important in heat exchange on such surfaces. However, there are presently physical and mathematical difficulties attendant to allowing for the effect of this section on heat exchange in evaporation in falling liquid films, particularly in theoretical investigations.

Heat transfer on the thermal initial section in the evaporation of turbulent falling liquid films was studied theoretically in [2]. Here, the investigators examined several models for the turbulent liquid, being an arbitrary combination of the familiar Van Drist and Shablevskii relations. The modification consisted mainly of allowing for the decay of eddy viscosity with approach to the free surface of the film. Otherwise, the theoretical Nu numbers significantly exceed the empirical values [3, 4]. It was established that the most satisfactory data is obtained using the model in [3] for the internal region of the film and modifying this model with the relation in [5] for the external region. Thus, only a numerical solution of the energy equation was obtained for several Re and Pr numbers, which complicates further use of the results.

Obtained below is an approximate analytical solution of a problem of heat transfer on the thermal initial section in the evaporation of turbulent liquid films flowing down a vertical surface. Here, we use the eddy viscosity model of Millionshchikov [6, 7] for the entire film thickness. The model gives a zero value for eddy viscosity on the free surface of the film. As already noted, this position is in accord with current representations. It should be mentioned that the use of this model of turbulence in the case of heat exchange without evaporation [8] produces results in agreement with the experimental data.

We will assume the physical properties of the liquid to be constant. The flow is hydrodynamically stabilized, with a flat free surface. The velocity profile in the film is described by a power relation with the exponent 1/7.

Given these assumptions, the evaporation heat-transfer problem reduces to the solution of the differential equation [8]

$$\eta^{1/7} \frac{\partial\theta}{\partial\xi} = \frac{\partial}{\partial\eta}\left[\left(1+\frac{\nu_t}{\nu}\frac{\mathrm{Pr}}{\mathrm{Pr}_t}\right)\frac{\partial\theta}{\partial\eta}\right] \tag{1}$$

with the conditions

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$$\theta(0,\eta)=0,\quad \partial\theta(\xi,0)/\partial\eta=-1,\quad \theta(\xi,1)=0. \tag{2}$$

We have the following relations for the distribution of kinematic eddy viscosity, in accordance with [6, 7]:

$$\nu_t/\nu=0,\quad \eta\leqslant\delta_0^+/\delta^+,\quad \nu_t/\nu=0.39(\eta\delta-\delta_0^+)(1-\eta),\quad \eta>\delta_0^+/\delta^+. \tag{3}$$

Two values, 7.8 and 11.6, are used for the dimensionless thickness of the viscous sublayer in the **calculations below.** The value of 7.8 was recommended in [6], while the value of 11.6 is normally used in boundary-layer theory. We will take 1 for the Prandtl number Pr_t.

The method in [9] will be used to find an approximate analytical solution of the problem (1), (2). We replace the left side of (1) by its mean value, i.e., we put

$$\int_0^1 \eta^{1/7}\frac{\partial\theta}{\partial\xi}d\eta=\frac{7}{8}\frac{d\theta_{av}}{d\xi}=\frac{\partial}{\partial\eta}\left[\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)\frac{\partial\theta}{\partial\eta}\right]. \tag{4}$$

Integrating (4) twice, with allowance for (2) we have

$$\theta=\theta_w+\frac{7}{8}\frac{d\theta_{av}}{d\xi}\int_0^\eta\frac{\eta d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)}-\int_0^\eta\frac{d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)}. \tag{5}$$

Satisfying the third condition of (2), we find from (5) that

$$\theta_w=\int_0^1\frac{d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)}-\frac{7}{8}\frac{d\theta_{av}}{d\xi}\int_0^1\frac{\eta d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)},$$

or in expanded form

$$\theta_w=\eta_0-\frac{7}{16}\frac{d\theta_{av}}{d\xi}\eta_0^2+n\left(2-\frac{7}{8}\frac{b}{\delta^+}\frac{d\theta_{av}}{d\xi}\right)/(0.39\mathrm{Pr}\,m), \tag{6}$$

$$m=\sqrt{b^2-4ac},\quad b=\delta^++\delta_0^+,\quad a=-\delta^+,\ c=-\left(\delta_0^+-\frac{1}{0.39\,\mathrm{Pr}}\right),\qquad n=\ln\left|\frac{(\delta^+-\delta_0^+)+m}{(\delta^+-\delta_0^+)-m}\right|. \tag{7}$$

Substitution of (6) into (5) finally gives us

$$\theta=\int_\eta^1\frac{d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)}-\frac{7}{8}\frac{d\theta_{av}}{d\xi}\int_\eta^1\frac{\eta d\eta}{\left(1+\frac{\tau_t}{\nu}\mathrm{Pr}\right)}. \tag{8}$$

Calculating the mean temperature of the liquid, we have

$$\theta_{av}=\frac{8}{7}\int_0^1\eta^{1/7}\theta d\eta=\frac{8}{7}\int_0^1\eta^{1/7}\left[\int_\eta^1\frac{d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)}\right]d\eta$$
$$-\frac{d\theta_{av}}{d\xi}\int_0^1\eta^{1/7}\left[\int_\eta^1\frac{\eta d\eta}{\left(1+\frac{\nu_t}{\nu}\mathrm{Pr}\right)}\right]d\eta=A_1-A_2\frac{d\theta_{av}}{d\xi}. \tag{9}$$

Integration of (9) with the initial condition $\theta_{av}(0) = 0$ gives

$$\theta_{av}=A_1\left[1-\exp\left(-\frac{1}{A_2}\xi\right)\right], \tag{10}$$

in which the constants A_1 and A_2 are determined from the following relations:

$$A_1=\frac{7}{15}\eta_0^{15/7}+\frac{(1+7b/\delta^+)n}{1.56\mathrm{Pr}\,m}, \tag{11}$$

$$A_2=\frac{49}{176}\eta_0^{22/7}+\frac{7b^2n}{3.12\,\mathrm{Pr}\,\delta^{+2}m}+\frac{1}{0.78\,\mathrm{Pr}\,\delta^+}\left[\frac{7}{8}(1-\eta_0^{8/7})+\right.$$
$$\left.+\frac{7}{4}\left(\frac{b^2-2ac}{\delta^+m}-\frac{b^2}{2\delta^+m}\right)n-\frac{7}{4}(1-\eta_0)-\frac{7}{8}(1-\eta_0^{8/7})\ln(0.39\mathrm{Pr})\right]. \tag{12}$$

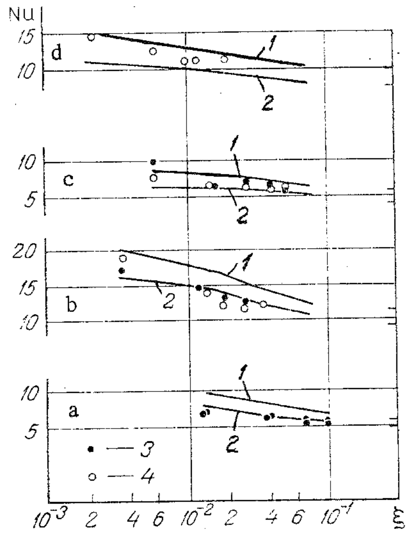

Fig. 1. Comparison of local Nusselt numbers on the thermal initial section. Calculation: 1) for $\delta_0^+ = 7.8$; 2) 11.6; experiment: 3) [2]; 4) [10]; a) Pr = 1.77; Re = 1900; b) 1.77; 4375; c) 5.7; 1045; d) 5.7; 2025. The notches at the right denote asymptotic values.

We find the local Nu numbers from the relation

$$\mathrm{Nu} = -\frac{\partial\theta(\xi,\ 0)/\partial\eta}{\theta_w} = \left\{\eta_0 - \frac{7}{16}\frac{A_1}{A_2}\eta_0^2 \exp\left(-\frac{1}{A_2}\xi\right) + n\left[2 - \frac{7}{8}\frac{A_1}{A_2}\frac{b}{\delta^+}\exp\left(-\frac{1}{A_2}\xi\right)\right]/(0.39\mathrm{Pr}\,m)\right\}. \tag{13}$$

Examination of Eq. (13) shows that, strictly speaking, it is invalid in the region $\xi \to 0$, since we obtain finite values of the Nu number at $\xi = 0$. However, in connection with the fact that this region is small, this consideration is not important for engineering calculations. At the same time, Eq. (13) exactly ensures a transition to asymptotic values of the Nu number on the section of stabilized heat exchange. This statement is supported below by the calculations.

Setting $d\theta_f/d\xi = 0$ in (7), we find the Nusselt number corresponding to the section of stabilized heat exchange:

$$\mathrm{Nu}_\infty = [\eta_0 + 2n/(0.39m\,\mathrm{Pr})]^{-1}. \tag{14}$$

Let us compare the calculations with (13) with the experimental data in [2, 10] obtained on the thermal initial section. Figure 1 shows results of comparison of the local Nusselt number for four cases. It can be seen that there is satisfactory agreement along the entire length of the thermal initial section for both $\delta_0^+ = 7.8$ and $\delta_0^+ = 11.6$. It should be noted that somewhat better agreement is seen for $\delta_0^+ = 7.8$ with Pr = 5.7, while the agreement for $\delta_0^+ = 11.6$ is improved with Pr = 1.77. The results obtained here are close to the results of the numerical solution proposed in [2] for model 3 and agree best with the experimental results. This pertains most of all to the length of the thermal initial section and the asymptotic values of the Nusselt number. In calculations with (13), the mean thickness of the film was calculated from the relation in [11]

$$\delta = 0.303\,(\nu^2/g)^{1/3}\,\mathrm{Re}^{7/12}, \tag{15}$$

from which it follows that

$$\delta^+ = 0.166\delta\,\mathrm{Re}^{7/8}. \tag{16}$$

In conclusion, we should note that the first experimental point corresponds to the longitudinal-coordinate value $x \approx 0.026$ m.

NOTATION

$(t - t_s)/(q\delta/\lambda)$, dimensionless temperature; t_s, saturation temperature; q, heat flux; x, y, longitudinal and transverse coordinates; $\xi = x/(\delta\mathrm{Pe})$, $\eta = y/\delta$, dimensionless coordinates; δ, mean thickness of film; $\mathrm{Pe} = \mathrm{Re}_1\mathrm{Pr}$, Peclet number; $\mathrm{Re}_1 = U_1\delta/\nu$, $\mathrm{Re} = U_{av}\delta/\nu$, Reynolds numbers of the liquid; $Pr = \nu/\alpha$, Prandtl number; U, velocity of liquid; ν, kinematic viscosity of the liquid; $\delta^+ = U_*\delta/\nu$, $\delta_0^+ = U_*\delta_0/\nu$, dimensionless thickness of the film and viscous sublayer; $\eta_0 = \delta_0^+/\delta^+$; U_*, absolute viscosity; $Nu = \alpha\delta/\lambda$, local Nusselt number; λ, a, thermal conductivity and diffusivity of the liquid; α, local heat-transfer coefficient. Indices: w, wall; av, mean; t, turbulent; l, free surface of the liquid.

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EFFECT OF DEFECTS IN THE POROUS SURFACE ON THE THERMOPHYSICAL CHARACTERISTICS OF HEAT-PIPE WICKS

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Results are presented from a study of the effect of defects in the porous surface of a heat-pipe wick on the capillary head developed by the structure.

It is currently thought [1] that the presence of a defect (pores of a diameter greater than other pores) in the porous surface of a heat-pipe wick causes a significant deterioration in the performance of the pipe. This statement is assumed to apply to wicks of any geometry.

Let us examine a meniscus located in a pore of a capillary structure in the form of a perforated screen positioned with a gap relative to the heating surface of a heat pipe (Fig. 1). If the gap is large, a meniscus with a spherical surface (Fig. 1a) will be formed in the pore. As shown in [2], the minimum radius is determined as follows in the general case for a pore with curvilinear generatrices:

$$R_{s_1}^0 = \frac{a + R(1 - \sin\varphi^0)}{2\sin(\varphi^0 - \Theta)}. \tag{1}$$

The relation for determining φ^0 was presented in [2]. For a cylindrical pore, R = 0 and $\varphi^0 = \pi/2$.

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